



Theoretical and experimental FUZZY modelling of building thermal dynamic response

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Abstract

In this paper the main advantages and disadvantages of two different types of modelling: theoretical and experimental are presented and discussed. The theoretical modelling is based on energy balances, which gives the overall model described by differential equations. On the basis of developed theoretical model a complex simulator in the MATLAB-Simulink environment was implemented. The second part is devoted to experimental modelling. In this paper a fuzzy model represented by non-linear relations between input and output variables obtained by least-squares optimisation method is investigated. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The procedure of model development and utilisation is in close relationship with simulation throughout the whole course of the cycle [1]. Modelling is an iterative and interactive procedure where cycling in the numerous local and global loops is often necessary. Such a procedure has evolutionary characteristics in the sense that it includes the course of model generation from speculations, hypotheses, and general model forming to the final simplified specific model usage. Here the transformation of qualitative information to quantitative data must be made. The cyclical procedure of modelling and simulation is performed by the aid of computer. The whole cycle, which passes from the real system through model building to the formal models and then through model utilisation back to the real system can be basically, divided into two main parts. The first one includes the phases of model building while the second part represents the analyses and interpretation of the model according to the real process. Models can be divided into many types. One of the possible classifications can distinguish physical, symbolic and mental models. Symbolic models are frequently used because they

are less problematic to manipulate than physical and mental models. They can be further divided into mathematical and nonmathematical models. The latter can be either linguistic, graphic or schematic. They have the common property, which is often very problematic to obtain precise information from them, especially from verbally expressed models. From many reasons mathematical models are the most suitable and the most widely used category of models. They are concise, unambiguous and uniquely interpretable, while their manipulation and the evaluation of alternatives are relatively inexpensive. A mathematical model can be defined as a mapping of relationship between physical variables of a system to be modelled into corresponding mathematical structures. Mathematical modelling can be further divided into theoretical and experimental modelling. The essence to theoretical modelling lies in the decomposition of the studied system into particular subsystems, which must be as simple as possible. The corresponding relations between chosen subsystems must then be determined on the basis of different balance equations and physical laws for the area under investigation. In the case of technical systems modelling, the known mass, energy and momentum balances are most frequently used, which gives the overall model expressed by differential equations. The basic principle of experimental modelling lies in the definition of the system inputs and outputs and the measurement of input and

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output signals, which enables the corresponding model generation. Using this approach the structure and parameters of the model which give responses that are equal, or as alike as possible, to the measured outputs of the system, using the same input signals, must be determined.

Only the input–output relations are interesting here and no information about the mechanisms, which cause these relations, can be obtained. Models obtained by experimental modelling can be either given in a form of differential or difference equations or can be fuzzy models given by relations between input and output signals in a verbal form using if-then statements or neuro models given with the structure of neural nets. Prior to firm about the final version of the formal or simulation model and before experimentation with it, the procedure of model verification and model validation must be performed. The term validation is concerned with demonstrating that the model is an adequate representation of reality, whereas the term verification involves checking the design consistency. In short, the fact that the model works as it was proposed has to be proved.

In this paper two different approaches to the modelling are developed and investigated. In Section 2 the development of mathematical model of thermal behaviour in building is presented and the structure of simulator, which was developed in MATLAB-Simulink simulation environment is shown. In Section 3 the basic features of fuzzy modelling are shown and the fuzzy model of the test rig “KAMRA” is described and developed as well.

The described modelling and simulation approaches were investigated with different final model goals so the comparison will be described in the sense of general advantages and drawbacks obtained from many different situations, measurements, simulation studies and experiences as well. So the presented different methodologies can only partly be regarded as alternatives for similar problems solutions. The paper is an overview of the modelling and simulation activities of our group in the field of thermal dynamics with the most important aim to implement an efficient control system, which is currently a very actual research topic [2,3].

2. Theoretical modelling of thermal behaviour in building and simulator KAMRA

Although the properties of the envelope are treated as time-independent parameters in most of all thermal simulation programs for the buildings, they are variable by their own nature. The variable nature is especially worth for the openings in building envelope. Frequently, openings are equipped with different less or more sophisticated shading systems, which enables in at least changing the shading ratio of opening or even their geometry. In some cases high-tech glazing (electrochromic, fotochromic, etc.) are used, where optical characteristics of glass could be

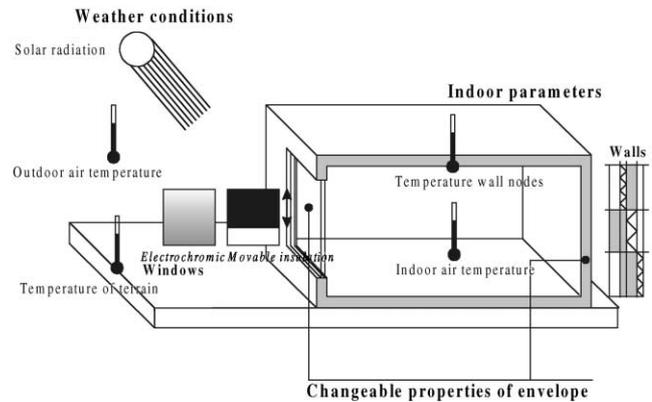


Fig. 1. Scheme of modelled system.

changed. Usually, these systems are manipulated manually by ‘users of the building’; due to assuring appropriate indoor thermal and light living conditions. On the other hand, the automatically adapting of such envelope properties appears as a new great opportunity of indirect controlling of the indoor living space parameters according to the current outdoor conditions (Fig. 1).

The developed simulator is an attempt in composing a mathematical model and a simulation tool, which has to offer the possibility of changing and also controlling the envelope properties. The theoretical model of the dynamic thermal response of the building, which includes also the possibility of time-dependent changing of envelope properties, is discussed in this section [4]. The described equations present the core of the extended mathematical model in which many additional features were included. In the mathematical model, material and geometrical properties of opaque and transparent elements of the envelope are no longer presented only as constant parameters, since their values could be changed during the 24 h simulation cycles. Simulator KAMRA is designed in MATLAB-Simulink environment. Using the simulator KAMRA different control strategies were designed, compared and validated before control system implementation [5].

2.1. Main features of the simulator KAMRA

The inputs to the simulation model are the outside conditions as well as dynamical parameters of the envelope.

Variable outdoors (weather) conditions:

- the outdoor air temperature,
- the temperature of the terrain,
- global solar radiation,
- level of cloudiness,
- ratio of diffuse/direct radiation.

Changeable properties of the building’s envelope:

- the opaque elements; thermal capacity and resistance of these elements can be changeable,

- the transparent elements (windows); geometry of openings, optical characteristics of glass and resistance of fill between glass panes are variable,
- interior properties; absorption, emission coefficients of walls and thermal capacity of furnishing are variable,
- other characteristic: changeable orientation.

Additional heating and cooling: the power of heater and ventilator.

The outputs of the simulation model are:

- the indoor air temperature and interior heat flow,
- the walls, windows and surface temperatures.

Main features of the simulator are:

- The possibility to simulate rectangular building with arbitrary walls, floor and ceiling composition.
- The opaque elements of the building envelope are floor, ceiling, walls and they are composed of 5 layers, which enables adequate thermal description of different envelope structures.
- In each wall one window of rectangular shape could be placed. All windows in the model are supposed to be double-glazed and filled with different gases.
- The inner space of the building can contain furniture and equipment. The ratio of furnishing/surface of the envelope is flexible, the material properties of furniture are optional.
- The solar radiation is composed of direct radiation and diffuse solar radiation. The ratio direct/diffuse radiation (DDR) in the model is flexible.

The level (CLD) of cloudiness is also an attribute of the outside conditions, as it affects the final amount of direct ($q_{\text{sol_e_dir}}$) and diffuse ($q_{\text{sol_e_dif}}$) radiation.

$$q_{\text{sol_e_dir}} = \text{DDR}(1 - \text{CLD})q_{\text{sol_e}}$$

$$q_{\text{sol_e_dif}} = q_{\text{sol_e}} - q_{\text{sol_e_dir}} \quad (1)$$

- The orientation of the building is optional parameter and it is defined by the declination angle between real (geographic) south and the direction of buildings axes.

The following suppositions are considered in the mathematical model:

- The whole mass of the inner air is supposed to have uniform temperature. In reality, the temperature of the air in the inner space is position-dependent function, but the temperature we used in calculation is an average temperature of the whole air mass.
- Temperature changing in directions along the wall or window surfaces are neglected, thus the conduction problems through the envelope elements can be treated as one-dimensional crosswise through them.
- Whole mass of furnishing/equipment is heated only by the surrounding air.

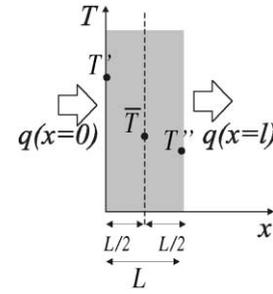


Fig. 2. Scheme of heat conduction through one material layer.

2.2. Thermal conduction

The problem of thermal conduction through the wall or window is treated as one dimensional. The basic assumption of the model is that the temperature changes in direction perpendicular to the walls or windows are much greater than the temperature changes within the surface area as shown in Fig. 2. The basic equation of thermal conduction (for one dimension) in a solid material can be simplified, if an average temperature of whole material layer is defined in Eq. (2).

$$\bar{T} = \frac{1}{L} \int_0^L T(x, t) dx. \quad (2)$$

Using this assumption the partial differential equation of heat conduction becomes an ordinary differential equation. The average temperature of a homogenous material layer is only time dependent as it is explained in Eq. (3) where T' and T'' are temperatures on the material borders, q is the heat-flow, T is an average temperature, ρ is specific density, c is specific heat of layer material and λ is thermal conductance.

$$\begin{aligned} \rho c \frac{\partial T}{\partial t} &= \lambda \frac{\partial^2 T}{\partial x^2}, \quad T = T(x, t), \\ S \rho \frac{\partial}{\partial t} \int_0^L T(x, t) dx &= S \lambda \int_0^L \frac{\partial^2 T(x, t)}{\partial x^2} dx, \\ S \rho c \frac{\partial}{\partial t} \int_0^L T(x, t) dx &= S \lambda \left[\frac{\partial T(x, t)}{\partial x} \Big|_{x=L} - \frac{\partial T(x, t)}{\partial x} \Big|_{x=0} \right] \\ &= S[q_{x=0} - q_{x=L}]. \end{aligned} \quad (3)$$

Finally, the last equation could be approximated, if the point of average temperature is supposed to be placed in the middle of the layer, in the form

$$S \rho c L \frac{d\bar{T}}{dt} = S \lambda \left[\frac{T' - \bar{T}}{L/2} - \frac{\bar{T} - T''}{L/2} \right]. \quad (4)$$

According to the expression in Eq. (4), it is possible to define thermal resistance R and thermal capacity C per unit area, for each layer of the wall using formulas

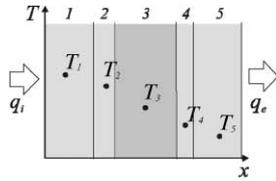


Fig. 3. Scheme of heat conduction through wall composed of five material layers.

in Eq. (5).

$$C = \rho cL, \quad R = \frac{L}{2\lambda}. \quad (5)$$

The dynamic of the average temperature in a homogenous layer is given by Eq. (5).

$$C \frac{d\bar{T}}{dt} = \frac{1}{R}(T' - \bar{T}) - \frac{1}{R}(\bar{T} - T''). \quad (6)$$

The model of heat conduction was derived for the wall composed of five material layers (Fig. 3). The heat flow on the interior surface of the wall is q_i , which is caused by radiant and convective heat flow. The dynamic changing of temperatures of layers T_i and layer border temperatures T'_i, T''_i is calculated using the set of differential and algebraic equations in Eqs. (7). R_i represents the thermal resistance and C_i is the capacity of layer indexed i .

$$\frac{d}{dt} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} \frac{1}{C_1(R_1 + R_2)} & \frac{1}{C_1(R_1 + R_2)} & 0 & 0 & 0 \\ \frac{1}{C_2(R_1 + R_2)} & -\frac{R_1 + R_2 + R_3}{C_2(R_1 + R_2)(R_2 + R_3)} & \frac{1}{C_2(R_2 + R_3)} & 0 & 0 \\ 0 & \frac{1}{C_3(R_2 + R_3)} & -\frac{R_2 + R_3 + R_4}{C_3(R_2 + R_3)(R_3 + R_4)} & \frac{1}{C_3(R_3 + R_4)} & 0 \\ 0 & 0 & \frac{1}{C_4(R_3 + R_4)} & -\frac{R_3 + R_4 + R_5}{C_4(R_3 + R_4)(R_4 + R_5)} & \frac{1}{C_4(R_4 + R_5)} \\ 0 & 0 & 0 & \frac{1}{C_5(R_4 + R_5)} & \frac{1}{C_1(R_4 + R_5)} \end{bmatrix} + \begin{bmatrix} \frac{q_i}{C_1} \\ 0 \\ 0 \\ 0 \\ -\frac{q_e}{C_5} \end{bmatrix} \quad (7)$$

Eq. (8) describes the dynamics of heat transfer through the window, where T_1 stands for the temperature of the inner crown-glass of the window and T_2 for outside glass pane. α_{wn} is conduction coefficient of air fill between inside and outside crown-glass surface. The solar radiant flows on the inside and outside crown-glass are described by q_{i_wn} and q_{e_wn} . The capacities of the inner and outside glass are C_1 and C_2 , respectively (Fig. 4).

$$\frac{d}{dt} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} -\frac{\alpha_{wn}}{C_1} & \frac{\alpha_{wn}}{C_1} \\ \frac{\alpha_{wn}}{C_2} & -\frac{\alpha_{wn}}{C_2} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} + \begin{bmatrix} \frac{q_{i_wn}}{C_1} \\ -\frac{q_{e_wn}}{C_2} \end{bmatrix}. \quad (8)$$

2.3. Thermal convection

The phenomena of convection in the model are reduced to the calculation of the values of conduction coefficients

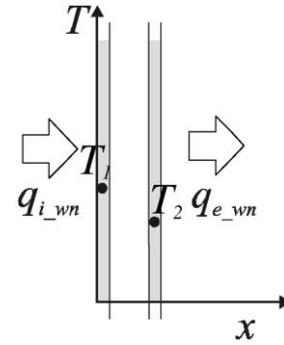


Fig. 4. Scheme of heat conduction through window.

in the boundary layer of the air. An empirical expression in Eq. (9), which defines the conduction coefficients of the external air layer α_e , which depend on the wind velocity v outside (measured in m/s), is

$$\alpha_e = 4.1v + 5.7. \quad (9)$$

Conduction coefficients of internal boundary air layer for the vertical surfaces (walls) depend on the surface temperatures, temperature of the indoor air and height of the wall in Eq. (10), where T is the surface temperature and T_{i_air} is the temperature of indoor air.

$$\alpha_i = 1.2|(T_{i_air} - T)|^{0.29+0.14h}. \quad (10)$$

Eqs. (11) are used for calculating values of conduction coefficients for horizontal surfaces (ceiling and floor). Note that the formulas for conduction coefficients of internal boundary air layer are only approximate, as one of the basic assumptions of the model is the uniformity of the temperature of the whole indoor air mass.

$$\begin{aligned} \text{if } T_{i_air} < T, \quad \alpha_i &= 1.0|(T_{i_air} - T)|^{0.33}, \\ \text{if } T_{i_air} > T, \quad \alpha_i &= 1.9|(T_{i_air} - T)|^{0.33}. \end{aligned} \quad (11)$$

2.4. Radiation

Solar radiation represents the influence of sunrays through transparent parts of the envelope, i.e. through glass apertures. The incident rate of the global solar radiation

flow $q_{\text{sol-e}}$ is partly reflected partly absorbed and partly transmitted (refracted) through the glass. The transmitted part of the solar radiation $q_{\text{sol-tr2}}$ is the solar heat flow that enters the room. The absorbed part of the solar radiation is partly absorbed on the external glass pane $q_{\text{sol-ab1}}$ and partly on the internal glass pane $q_{\text{sol-ab2}}$

$$q_{\text{sol-tr2}} = q_{\text{sol-e}} \left[\text{TR}_2 e^{\frac{\ln(\text{TR}_1(n_1^2+1)/2n_1)}{\cos \vartheta_1}} e^{\frac{\ln(\text{TR}_2(n_2^2+1)/2n_2)}{\cos \vartheta_2}} \right],$$

$$q_{\text{sol-ab1}} = q_{\text{sol-e}} \left[\text{TR}_1 \left(1 - e^{\frac{\ln(\text{TR}_1(n_1^2+1)/2n_1)}{\cos \vartheta_1}} \right) \right],$$

$$q_{\text{sol-ab2}} = q_{\text{sol-tr1}} - q_{\text{sol-tr2}}. \tag{12}$$

When calculating the coefficients of transmission and absorption, we took into account that all windows in the model are double-glazed. n_1 and n_2 are the indices of refraction of the external and internal glass pane, respectively. A_1 and A_2 are the corresponding coefficients of reflection (albedo). TR_1 is the coefficient of transmission of the external glass pane if the angle of incidence is zero, and TR_2 is the coefficient of transmission of complete glazing, respectively. TR is the coefficient of transmission of the complete glazing in the case of angle of incidence ϑ .

$$\vartheta_1 = \arcsin\left(\frac{\sin \vartheta}{n_1}\right),$$

$$\vartheta_2 = \arcsin\left(\frac{\sin \vartheta}{n_2}\right),$$

$$A_1 = \frac{1}{2} \left[\frac{\sin^2(\vartheta - \vartheta_1)}{\sin^2(\vartheta + \vartheta_1)} + \frac{\tan^2(\vartheta - \vartheta_1)}{\tan^2(\vartheta + \vartheta_1)} \right],$$

$$A_2 = \frac{1}{2} \left[\frac{\sin^2(\vartheta - \vartheta_2)}{\sin^2(\vartheta + \vartheta_2)} + \frac{\tan^2(\vartheta - \vartheta_2)}{\tan^2(\vartheta + \vartheta_2)} \right],$$

$$\text{TR}_1 = \left[\frac{(1 + A_1)(1 + A_2)}{(1 + A_1)(1 + A_2) - 4A_1A_2} \right],$$

$$\text{TR}_2 = \left[\frac{(1 + A_1)(1 + A_2)}{(1 + A_1)(1 + A_2) - 4A_1A_2} \right] \left[\frac{1 - A_2}{1 + A_2} \right]. \tag{13}$$

The transmission coefficients are time dependent, while the angle of incidence ϑ takes on different values as the sun continuously changes its position. The angle of incidence is calculated for each orientation using the following formulas:

$$\omega = \left(0.5 - \frac{t}{86400}\right) 360,$$

$$\delta = 23.45 \sin\left(360 \left(\frac{284 + \text{IoD}}{365}\right)\right),$$

$$\alpha = \arcsin[\sin \varphi_0 \sin \delta + \cos \varphi_0 \cos \delta \cos \omega],$$

$$\varphi = \arcsin\left[\frac{\sin \omega \cos \delta}{\cos \alpha}\right],$$

$$\theta_{\text{south}} = \arccos[\cos \delta \sin \varphi_0 \cos \omega \cos \gamma - \sin \delta \cos \varphi_0 \cos \gamma + \cos \delta \sin \omega \sin \gamma],$$

$$\theta_{\text{north}} = 180^\circ - \theta_{\text{south}},$$

$$\theta_{\text{east}} = \arccos[\cos \delta \sin \omega \cos \gamma + \sin \delta \cos \varphi_0 \sin \gamma - \cos \delta \sin \varphi_0 \cos \omega \sin \gamma],$$

$$\theta_{\text{west}} = 180^\circ - \theta_{\text{east}}. \tag{14}$$

In Eqs. (14) γ means the orientation of the object (angle between building axis and the south direction), φ_0 means the geographical latitude, δ is the declination of sunrays according to the axis perpendicular to the plain of the object floor, ω is the angle in the horizontal plain and it presents the time measured with angle, φ is the angle between the horizontal projection of the direction of sunrays and the direction of the south, α is the angle between the incidence sunrays direction and its horizontal projection in vertical plain, t means time (s) and IoD index of the day in the year. Finally, θ is the angle of incidence to the window of appertaining orientation [6].

2.5. Long-wave radiation

Stefan’s law defines the radiation emitted isotropically by a surface, which is heated to a particular temperature and it is the same for all directions in the hemisphere above the observed surface. Other surfaces in the vicinity can intercept only part of the radiation that is emitted from the heated surface. The received radiant flow depends on the distance between the surfaces and the mutual position. The rate of received radiant flow is defined by the geometrical coefficient. In the case of special mutual position of parallel or perpendicular surfaces, as it occurs in the model, the geometrical coefficients can be evaluated analytically. The equations for evaluating the geometrical coefficients F_{ij} are given in Eq. (15).

$$q_{ij} = \varepsilon_i \varepsilon_j S_i F_{ij} (T_i^4 - T_j^4),$$

$$F_{ij} = \frac{2}{\pi XY} \left[0.5 \ln \left[\frac{(1 + X^2)(1 + Y^2)}{1 + X^2 + Y^2} + X \frac{\sqrt{1 + Y^2}}{\text{tg}(X/\sqrt{1 + X^2})} + Y \frac{\sqrt{1 + X^2}}{\text{tg}(Y/\sqrt{1 + Y^2})} - \frac{X}{\text{tg}(X)} - \frac{Y}{\text{tg}(Y)} \right] \right],$$

$$F_{ij} = \frac{2}{\pi Y} \left[\frac{Y}{\text{tg}(Y)} + \frac{X}{\text{tg}(X)} - \frac{\sqrt{X^2 + Y^2}}{\text{tg}(1/\sqrt{1 + X^2})} + 0.25 \ln \left[\frac{(1 + X^2)(1 + Y^2)}{1 + X^2 + Y^2} + X \frac{\sqrt{1 + Y^2}}{\text{tg}(X/\sqrt{1 + X^2})} + Y \frac{\sqrt{1 + X^2}}{\text{tg}(Y/\sqrt{1 + Y^2})} - \frac{X}{\text{tg}(X)} - \frac{Y}{\text{tg}(Y)} \right] \right],$$

$$X = \frac{a}{c}, \quad Y = \frac{b}{c}. \tag{15}$$

where X and Y stands for rates described in Fig. 5.

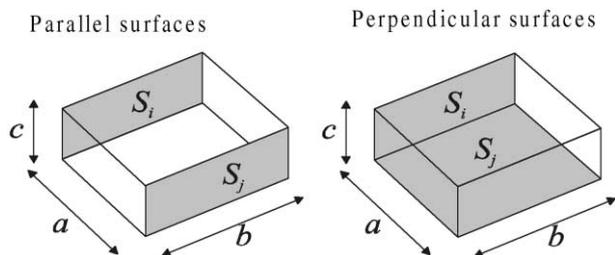


Fig. 5. Scheme of long-wave radiation between surfaces by mutual position of walls in the model.

2.6. The calculation procedure

Solar radiation penetrates to the interior through the windows. The amount of the direct solar radiation, which penetrates through a single window (indexed i), is defined in Eq. (16). TR_i stands for the transmission coefficient of window (i) for the direct solar radiation fit to the current value of the angle of incidence ϑ_i , DDR is the ratio of the diffuse and direct solar radiation, CLD is the level of cloudiness, P_{i_wn} is the area of single window i . The diffuse solar radiation, which penetrates through window (i), is defined by the Eq. (16).

$$q_{sol_dir_tr-i} = P_{i_wn} TR_{dir-i} \times \left[DDR(1 - CLD) q_{sol_e_dir} \frac{\cos(\vartheta_i)}{\cos(\varphi_0 - \delta)} \right],$$

$$q_{sol_dif_tr-i} = P_{i_wn} TR_{dif-i} \left[q_{sol_e_dif} - DDR(1 - CLD) q_{sol_e} \frac{\sin(\alpha)}{\cos(\varphi_0 - \delta)} \right]. \quad (16)$$

Because the sun constantly changes its position, different interior surfaces will be illuminated according to the current time and date, orientation and position of the window. Direct radiation can fall in one selected moment to one or more surfaces. Special functions calculate the ratio of solar heat flow that falls to particular surfaces. For example, if the corner of the room is illuminated, the radiation will be distributed to two walls and floor, as it is shown in Fig. 6.

The part of direct solar radiation, which is received by the single wall (i) is expressed with the ratio coefficient SS_i . The sum of all ratio coefficients SS_i, SS_j , and SS_k must be one. The amount of absorbed direct solar radiation can be defined by Eq. (17). i is the index of the observed wall, and j are indices of the particular window, through which the radiation penetrates. P_{i_wl} is the area of opaque wall P_{i_wn} is the area of window, which is placed in the wall i .

$$q_{ab_sol_dir-i} = \frac{P_{i_wl}}{P_{i_wn} + P_{i_wl}} \sum_j SS_i q_{sol_dir_tr-j} AB_j. \quad (17)$$

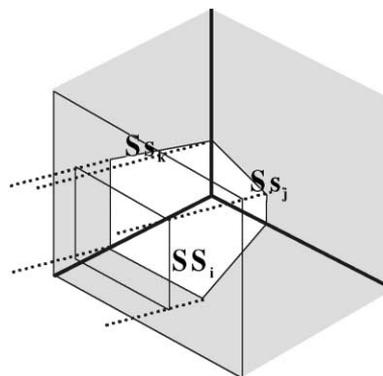


Fig. 6. Scheme of distribution of illuminated surfaces by direct solar radiation.

The diffuse solar radiation is treated similarly as long-wave radiation, using geometrical coefficients. The amount of diffuse solar radiation received by a single wall surface (indexed i) is presented with the Eq. (18), where j stands for the indices of windows. Finally, the whole primer heat-flow to the particular wall or window surface, which is caused by direct and the diffuse solar radiation is represented by

$$q_{sol_dif-i} = \frac{P_{i_wl}}{P_{i_wn} + P_{i_wl}} \sum_j F_{ij} q_{sol_dif_tr-j},$$

$$q_{sol-i} = q_{sol_ab_dir-i} + q_{sol_dif-i}. \quad (18)$$

The following contributions of solar heat-flow, which fall to a particular surface, are caused by the solar radiation, which is a reflected on particular wall. The number of reflections is set to 10, as it was empirically estimated that higher number of reflections does not significantly effect achieved temperatures and heat-flows. The reflected radiation depends on the absorption coefficient AB , which is an optical characteristic of the wall surface. The reflected radiation to the particular wall (i) is defined by Eq. (19), j are indices of other walls and windows.

$$q_{refl-i} = \sum_j q_j (1 - AB_j). \quad (19)$$

The absorbed heat flow causes increased temperatures of the surface, which is illuminated. The temperature differences between surfaces appear, and initiates long-wave radiation to the selected inner surface S_i , which is the sum of contributions of radiant flows from all the surrounding walls and windows, ceiling and floor, given by

$$q_{lw-i} = \sum_j S_j F_{ji} (T_j^4 - T_i^4). \quad (20)$$

The total heat flow to the particular wall or window caused by direct and reflected solar radiation and long-wave radiation is then

$$q_i = q_{sol-i} + q_{refl-i} + q_{lw-i}. \quad (21)$$

After estimating heat flows for single walls and windows, the calculation of temperatures is accomplished according

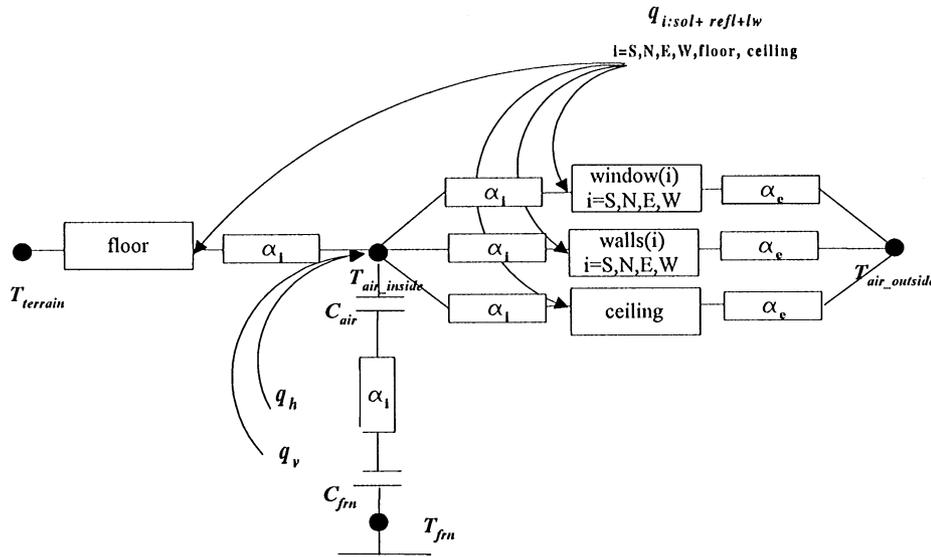


Fig. 7. Scheme of dynamical temperatures.

to Fig. 7. In this scheme blocks named walls or windows stand for appropriate element structure, as it was explained in Section 2.2. In Eq. (22), Q_h stands for the power of the heater and Q_v for the power of the cooler and C_{air} is the capacity of the inner air. An interesting option of the model is that some furniture or equipment can be installed in the room and can be heated (or cooled) by the surrounding air. This seems to be quite an important property of the inner space, because thermal capacity C_{fnn} of furniture is usually much greater than the thermal capacity of the indoor air.

$$\frac{dT_{air}}{dt} C_{air} = \left[\sum_j \alpha_{i-j} P_{j_wl} (T_{air} - T_j) + \alpha_{i-j} P_{j_wn} (T_{air} - T_j) \right] - \alpha_i (T_{air} - T_{fnn}) + Q_h + Q_v,$$

$$C_{fnn} \frac{dT_{fnn}}{dt} = \alpha_i (T_{air} - T_{fnn}). \quad (22)$$

The concept of the simulator is shown in Fig. 8. Thermal flows entering the room are calculated separately in a special block. In this block, functions for estimating the current sun position, optic characteristics depending on the angle of incidence and finally direct and diffuse solar radiation entering the room are gathered.

The thermal flows represent inputs to the thermal dynamics block. As already described, the thermal dynamics of the model is the result of heat conduction, convection, infrared radiation between surfaces with different temperatures and global solar radiation, which penetrates through openings and is partly absorbed and partly reflected on the interior surfaces. These phenomena are gathered in the essential part of the model, the so-called thermal dynamics block. In this part walls and windows are modelled on the basis of thermal resistance net approach and appropriately

combined into a complete model which can be described by a set of algebraic and differential equations. They are then combined by differential equation that describes the inside air temperature. Special block calculates the variations of building envelope properties. In the model only the geometry of window can be variable, hence it is possible to alternate optical and thermal conductivity characteristics of windows. As the calculations in the thermal flows and envelope properties are very complicated, the appropriate blocks work as discrete ones with user defined sampling interval. Between successive sampling instants the signals hold constant values. The appropriate sampling time is selected with regard to the requested accuracy and calculation efficiency. On the other hand, the calculations in thermal dynamics block are determined with so called step size which automatically adapts during simulation in order to assure the maximal calculation efficiency with regard to the error tolerance as well as the speed of simulation. The strategy of the step size adaptation depends on the selected numerical integration method as the basic numerical method in continuous simulation.

After the development of the mathematical model and the simulator concept (Fig. 8) the appropriate programming tool had to be selected. The most important requirements for the appropriate selection were as follows:

- Modular and transparent syntax. Model should be easy to understand and to modify as well.
- Modern graphic user interface should enable the unskilled users in modelling and simulation to efficiently experiment with the model. Users must concentrate to thermal problems instead of problems with modelling, simulation, programming, etc.
- High numerical accuracy and robustness.
- Fast simulation.

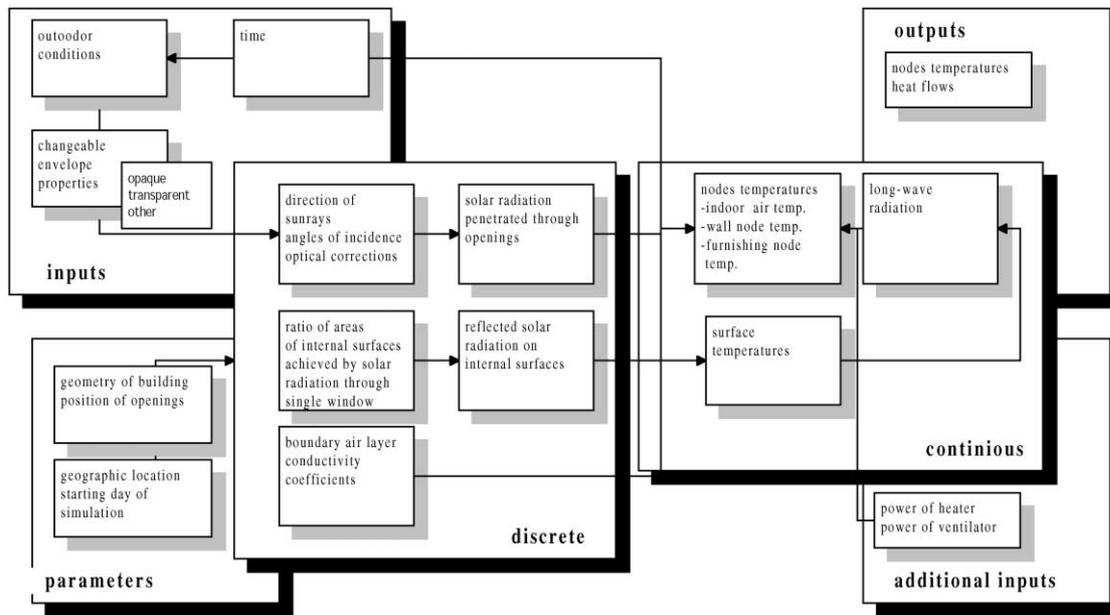


Fig. 8. Simulator concept.

- Portable models. The selected environment should be a widely spread one, used not only on academic institutions but also in industry. So developed models can be easily transferred between different computers, groups or institutions.
- With regard to the developed simulation concept the capability for the inclusion of continuous and discrete submodels into the simulation model must be presented.
- In the chosen environment different toolboxes must give powerful possibilities not only for simulation but also for analysis, design, graphical results presentation etc.
- If possible, control structures obtained by off line simulation and design can be automatically coded for appropriate target hardware giving efficient real time implementations [7,8].

Fig. 9 shows the screen outlook on the highest hierarchical level of the developed model. In the block *Initialisation* all the parameters about the materials, geometry of window, orientation, geographic location and starting simulation time are given.

So simulation of the behaviour in the case of different materials, orientations, geographic location, position and number of windows and period of the year can be performed. In the block *Outdoor temperature T_e and Solar radiation* the measured or predefined values of outdoor temperature, temperature of terrain, solar radiation, ratio direct/diffuse radiation and level of cloudiness are given in appropriate data files. Also the power of the heater and the ventilator are defined in block *Heating and Ventilating*. Position of the roller blind is described or defined in block *Geometry of openings*. In blocks *Opaque elements* variable properties can be defined or generated

(thermal capacity and resistance), in block *Transparent elements* properties of openings (optical prop. and thermal resistance), respectively. These blocks represent the input variables of the model. The output variable T_{mp} of the model is vector, where temperatures and heat-flows are gathered. But the simulator can be modified easily in the sense that also other variables of the model can be monitored.

2.7. The evaluation of the simulator

The geometry of window size was chosen to be variable in the experiment. Through series of experiment we tried to investigate the thermal response of object, when the geometry is alternating. The test chamber is a box with all dimensions 1 m (Fig. 10). The south wall is completely glazed, double-glazing is composed of two layers of standard clear glass and air fill, the thickness of wooden frame is 5 cm. The roller blind is as external PVC blind and the alternating window geometry was realised by moving the blind to desired position. Walls, floor and ceiling are composed of dry wall panel 1 cm, mineral wool 8 cm dry wall panel 2 cm (from outside). Internal walls are painted in light grey colour. The box is shifted off the ground and the roof is ventilated in order to avoid overheating caused by direct radiation on the roof. Measured values for outdoor conditions were global and reflected solar radiation and outdoor air temperature. Pyranometer CM-6B (Kipp & Zonnen delft BV) was used for measuring solar direct and reflected radiation. Thermocouples type T was used for measuring temperature. The temperature of indoor air defines thermal response of the object and it was

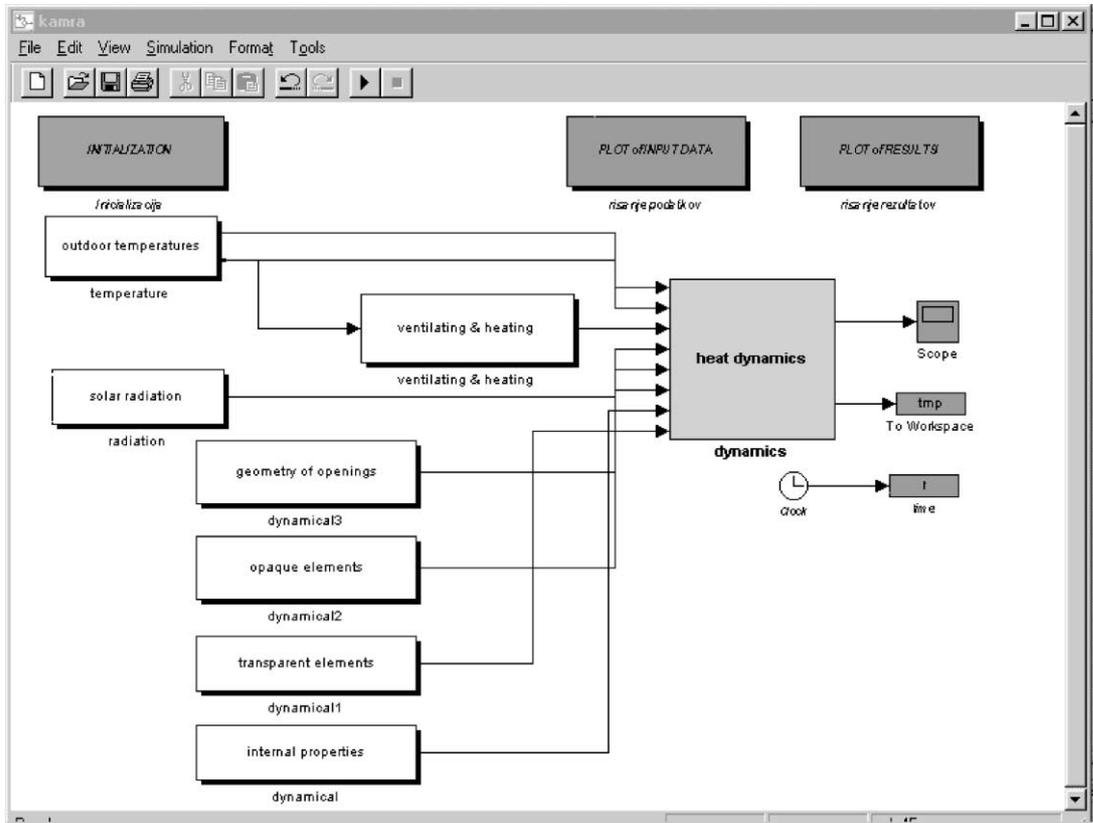


Fig. 9. The simulator in MATLAB-Simulink environment.



Fig. 10. Picture of testing cell.

also measured with thermocouples type T. Window size was expressed as ratio of shaded area and whole glazing area. For the purpose of collecting of different samples of the outdoor environment conditions, some series of measurements were executed in different seasons of the year. Position of blind was changed randomly in different time intervals independent of the outdoor conditions.

Typical charts (presented in Figs. 11 and 12) taken from experiments are showing the measured outdoor conditions (outdoor temperature and global radiation), the regime of

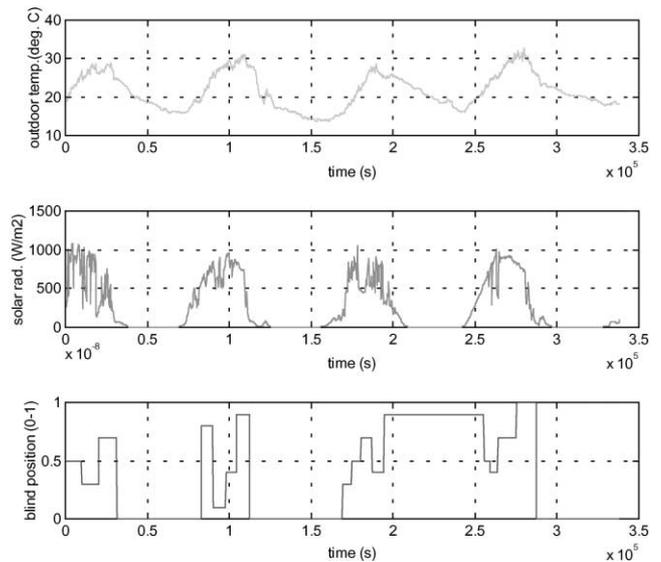


Fig. 11. Blind position, global solar radiation and outdoor temperature June 10, 1998.

blind moving (on interval [0,1], 0-blind does not cover the window, 1-whole window is covered with the blind) and the indoor air temperature as the ‘result’ of experiment. The first series of measurements presented in the paper started on June 10, 1998, the second started in October 12, 1998.

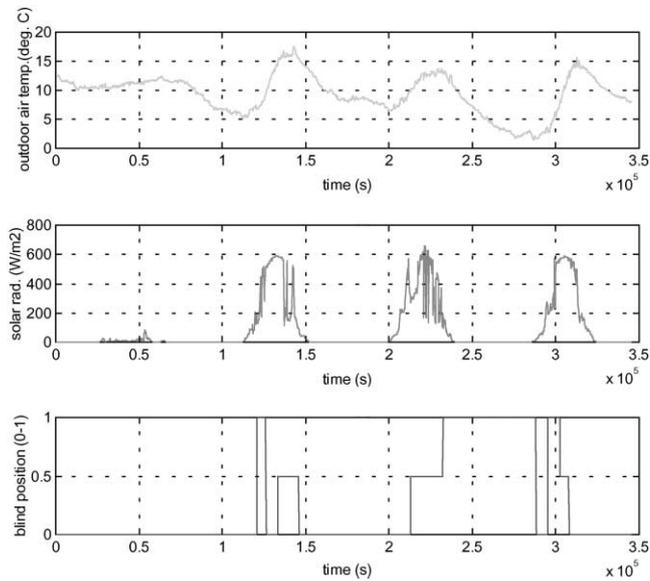


Fig. 12. Blind position, global solar radiation and outdoor temperature October 12, 1998.

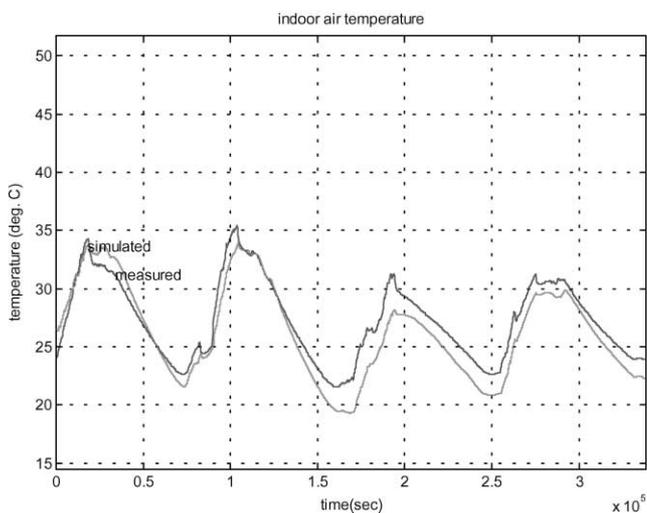


Fig. 13. Comparison of the measured and simulated indoor temperature June 10, 1998.

The validation of the model is one of the most important tasks in each modelling cycle. It is based on the comparison of the measured and simulated results. Several measurements were used for appropriate final parameters tuning of the theoretical model of the KAMRA. Another set of measurements was used for model validation. Simulations were obtained with the measured outdoor temperatures and global radiation as input variables taken from the experiments as well as the signal for blind moving regime. The comparison of the simulated indoor temperature and the measured one are presented in Figs. 13 and 14. The error between calculated and measured values is acceptable in the range of 5–20%. Mainly it is caused by unexpected ventilation heat-losses through some cracks in the dry wall panels and by the influence of wind.

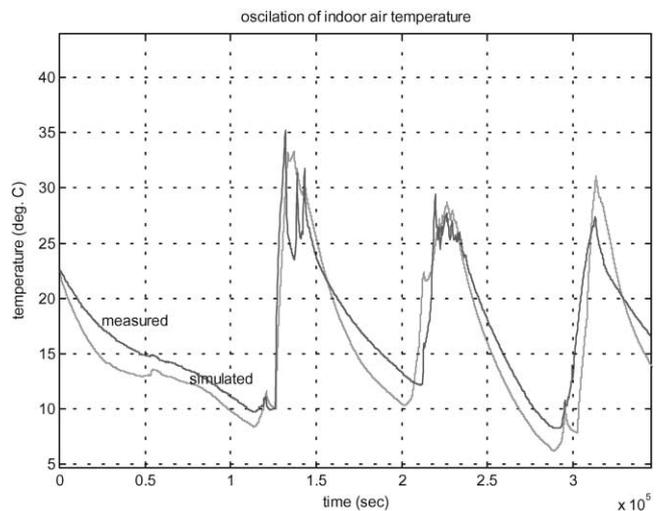


Fig. 14. Comparison of the measured and simulated indoor temperature October 12, 1998.

3. Experimental modelling based on fuzzy logic

In this section an attempt of experimental modelling of the described test rig “KAMRA” with fuzzy approach is shown. In previous work, a theoretical mathematical modelling was developed and validated. The heat flows were modelled by the aid of energy balance equations. According to many unknown phenomena and parameters several simplifications were introduced. The model parameters, which could not be measured, have been estimated experimentally comparing the experimental and simulation results. Theoretical modelling of thermal behaviour in the test “KAMRA” is a complicated and time-consuming task. That was the main reason to develop a non-linear fuzzy model of the same physical plant. This approach gives comparable results as obtained with theoretical modelling, but they can be obtained much faster and easier. However, the main drawback of this methodology is that the obtained model is valid only for the plant where the measurements were made and it cannot be extended to similar processes. Experimental model could be seen as a model, which has been learned on the basis of measurement, made on the same plant. So it cannot give results as relevant as in the case when the inputs to the model are very different from those which are used in the learning set. To obtain a good model for the whole working area, the measurements for the whole working area should be provided. This is the main disadvantage of this type of modelling. But it is not a real disadvantage in comparison to the theoretical model, which has to be validated for the whole working area.

Fuzzy model represents a non-linear mapping between input and output variables. Dynamic systems are usually modelled by feeding back delayed input and output sig-

nals. The common non-linear model structure is NARX (Non-linear AutoRegressive with eXogenous input) model, which gives the mapping between past input–output data and the predicted output. Fuzzy modelling or identification aims at finding a set of fuzzy if-then rules with well-defined parameters, that can describe the given I/O behaviour of the process. In the recent years many different approaches to fuzzy identification have been proposed in the literature [9]. In our case the model is based on modified Sugeno-type fuzzy model.

3.1. Basics of fuzzy sets

Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth — truth values between “completely true” and “completely false”. It was introduced in the 1960s as a means to model the uncertainty of natural language. Fuzzy theory should be seen as a methodology to generalise any specific theory from a crisp (discrete) to a continuous (fuzzy) form.

3.1.1. Fuzzy subsets

Just as there is a strong relationship between Boolean logic and the concept of a subset, there is a similar strong relationship between fuzzy logic and fuzzy subset theory.

In classical set theory, a subset U of a set S can be defined as a mapping from the elements of S to the elements of the set $\{0, 1\}$,

$$U : S \rightarrow \{0, 1\}. \quad (23)$$

This mapping may be represented as a set of ordered pairs, with exactly one ordered pair present for each element of S . The first element of the ordered pair is an element of the set S , and the second element is an element of the set $\{0, 1\}$. The value zero is used to represent non-membership, and the value one is used to represent membership. The truth or falsity of the statement

$$x \text{ is in } U \quad (24)$$

is determined by finding the ordered pair whose first element is x . The statement is true if the second element of the ordered pair is 1, and the statement is false if it is 0.

Similarly, a fuzzy subset F of a set S can be defined as a set of ordered pairs. Each with the first element from S , and the second element from the interval $[0, 1]$, with exactly one ordered pair present for each element of S . This defines a mapping between elements of the set S and values in the interval $[0, 1]$. The value zero is used to represent complete non-membership, the value one is used to represent complete membership, and values in between are used to represent intermediate *degrees of membership*. The set S is referred to as the *universe of discourse* for the fuzzy subset F . Frequently, the mapping is described as a function, the *membership function* of F . The degree to which the statement

$$x \text{ is in } F \Leftrightarrow x, \mu_F(x) \quad (25)$$

is true is determined by finding the ordered pair whose first element is x . The *degree of truth* of the statement is the second element of the ordered pair.

3.1.2. Logic operations

Now that we know what a statement like X is LOW means in fuzzy logic, how do we interpret a statement like

$$X \text{ is LOW and } Y \text{ is HIGH or (not } Z \text{ is MEDIUM)}. \quad (26)$$

The standard definitions in fuzzy logic are

$$\text{not}(x) = 1.0 - \mu(x),$$

$$x \text{ and } y = \min(\mu(x), \mu(y)),$$

$$x \text{ or } y = \max(\mu(x), \mu(y)). \quad (27)$$

Note that if just the values zero and one are plugged into these definitions, the same truth tables as for conventional Boolean logic are obtained. This is known as the *extension principle*, which states that the classical results of Boolean logic are recovered from fuzzy logic operations when all fuzzy membership grades are restricted to the traditional set $\{0, 1\}$. This effectively establishes fuzzy subsets and logic as a true generalisation of classical set theory and logic. In fact, by this reasoning all crisp (traditional) subsets are fuzzy subsets of this very special type; and there is no conflict between fuzzy and crisp methods.

Assume that a fuzzy subset HOT is defined by the membership function in analytical form

$$\text{HOT} : \mu(x) = \begin{cases} \text{if } x \leq 20^\circ\text{C}, & \mu(x) = 0, \\ \text{if } x > 20^\circ\text{C and } x < 30^\circ\text{C}, & \mu(x) = \frac{x - 20}{10}, \\ \text{if } x \geq 30^\circ\text{C}, & \mu(x) = 1. \end{cases} \quad (28)$$

The membership of the temperature $x = 24^\circ\text{C}$ to the set HOT results in membership value

$$\text{HOT} : x = 24^\circ\text{C}, \quad \mu_{\text{HOT}}(x) = 0.4. \quad (29)$$

In case the whole domain of physical variable should be fuzzified a fuzzy variable, which is represented by a collection or set of fuzzy subsets, is obtained. A new fuzzy variable which represents the indoor temperature TEMP can be formed from three different fuzzy subsets

$$\text{TEMP} = [\text{COLD}, \text{MEDIUM}, \text{HOT}], \quad (30)$$

where the new fuzzy subsets are defined as

$$\text{COLD} : \mu(x) = \begin{cases} \text{if } x \leq 10^\circ\text{C}, & \mu(x) = 1, \\ \text{if } x > 10^\circ\text{C and } x < 20^\circ\text{C}, & \mu(x) = \frac{20 - x}{10}, \\ \text{if } x \geq 20^\circ\text{C}, & \mu(x) = 0, \end{cases} \quad (31)$$

MEDIUM : $\mu(x)$

$$= \begin{cases} \text{if } x \leq 10^\circ\text{C}, & \mu(x) = 0, \\ \text{if } x > 10^\circ\text{C and } x \leq 20^\circ\text{C}, & \mu(x) = \frac{x - 10}{10}, \\ \text{if } x > 20^\circ\text{C and } x < 30^\circ\text{C}, & \mu(x) = \frac{30 - x}{10}, \\ \text{if } x \geq 30^\circ\text{C}, & \mu(x) = 0. \end{cases} \quad (32)$$

To define a certain indoor temperature as a fuzzy value, the membership values of all subsets in fuzzy set TEMP must be calculated. This will lead to fuzzy value of temperature x

$$\text{TEMP}(x) = [\mu_{\text{COLD}}(x), \mu_{\text{MEDIUM}}(x), \mu_{\text{HOT}}(x)], \quad (33)$$

which uniformly represents the crisp value. In our case the temperature $x = 28^\circ\text{C}$ is represented by fuzzy value or fuzzy vector

$$\mu_{\text{TEMP}}(x) = \text{TEMP}(x) = [0, 0.2, 0.8]. \quad (34)$$

The fuzzy sets and fuzzy values are of the main importance in the case of fuzzy identification, because in this case the input and output space are fuzzified in a fuzzy manner.

3.2. Fuzzy modelling

In this subsection Takagi–Sugeno fuzzy model is discussed [10–13]. Suppose the rule base of a fuzzy system is as follows:

$$\mathbf{R}_i: \text{if } x_1 \text{ is } A_i \text{ and } x_2 \text{ is } B_i \text{ then } y = f(x_1, x_2), \quad (35)$$

where x_1 and x_2 are input variables of the fuzzy system, y is an output variable, A_i, B_i are fuzzy sets characterised by their membership functions. The *if*-parts (antecedents) of the rules describe fuzzy regions in the space of input variables and the *then*-parts (consequent) are functions of the inputs usually defined as

$$f_i(x_1, x_2) = a_i x_1 + b_i x_2 + r_i, \quad (36)$$

where a_i, b_i are the consequent parameters. Such fuzzy model can be regarded as a collection of several linear models applied locally in the fuzzy regions defined by the rule antecedents. Smooth transition from one subspace to another is assured by the overlapping of the fuzzy regions.

The goal of the fuzzy modelling was to obtain the model, which can be used for indoor temperature control through the movable blind. It has to define the relation between the roller blind position P and indoor temperature T_c . The expected dynamics between these two signals is of the first order. So the fuzzy model is given in the form of the modified Sugeno model of the first order and can be determined with fuzzy rules of the following form:

$$\mathbf{R}_i: \text{if } x_1 \text{ is } A_i \text{ and } x_2 \text{ is } B_i$$

then

$$y(k+1) = a_i(x_1, x_2)y(k) + b_i(x_1, x_2)u(k) + r_i, \quad (37)$$

where $u(k)$ and $y(k)$ are the input and output variables of the process. $a_i(x_1, x_2)$ and $b_i(x_1, x_2)$ are model parameters expressed as functions of variable x_1 which is the rate of global solar radiation flow $q_s(k)$ and x_2 which is the outside temperature $T_e(k)$. The input of thermal model $u(k)$ is the position of the roller blind $P(k)$ and the output of the model is the indoor temperature $T_c(k)$. According to this the previous rule can be rewritten in the following form:

$$\mathbf{R}_i: \text{if } q_s \text{ is } A_i \text{ and } T_e \text{ is } B_i$$

then

$$T_c(k+1) = a_i(q_s, T_e)T_c(k) + b_i(q_s, T_e)P(k). \quad (38)$$

The main modification of Sugeno fuzzy model form is made in a sense that variables in antecedent part are not directly used in linear combination in consequent part of the rule. Those variables influence strongly on time constant and gain of the process model. Both parameters can be presented as follows:

$$a_i(q_s, T_e) = \mathbf{a}_f(q_s, T_e)\mu(q_s, T_e),$$

$$b_i(q_s, T_e) = \mathbf{b}_f(q_s, T_e)\mu(q_s, T_e), \quad (39)$$

where $\mathbf{a}_f(q_s, T_e)$ and $\mathbf{b}_f(q_s, T_e)$ are fuzzified parameters of the model. The fuzzy vector $\mu(q_s, T_e)$ is obtained by fuzzy intersection of fuzzy vectors $\mu(q_s)$ and $\mu(T_e)$ where fuzzy vector stands for a set of corresponding membership values. Fuzzified model parameters are obtained by least-squares method. In the case of the model with fuzzified parameters the regression vector consists of fuzzy vectors multiplied by values of input variables and can be presented by the following equation:

$$\varphi = [-T_c(k)\mu(q_s, T_e), \mu(q_s, T_e)P(k), 1]. \quad (40)$$

Indoor temperature can be according to the described regression vector written as

$$\hat{T}_c(k+1) = f(\varphi), \quad (41)$$

where function f represents a non-linear transformation between input and output domain. This transformation is linear in parameters, so the fuzzy identification means to find the fuzzified parameters of function f in the sense of least-squares error between the predicted indoor temperature \hat{T}_c and measured variable T_c .

According to the given transcription the model can be seen as the *model with fuzzified parameters*.

3.3. Results of experimental fuzzy modelling

Two different fuzzy models have been developed. The first one is simulation fuzzy model and the second one is one-step ahead prediction fuzzy model, which can be used for the control purposes. The process exhibits the dynamics, which can be described by a first- or a second-order model. Further investigations have shown that the proposed first-order fuzzy model structure give suitable results. The major problem in fuzzy model development was

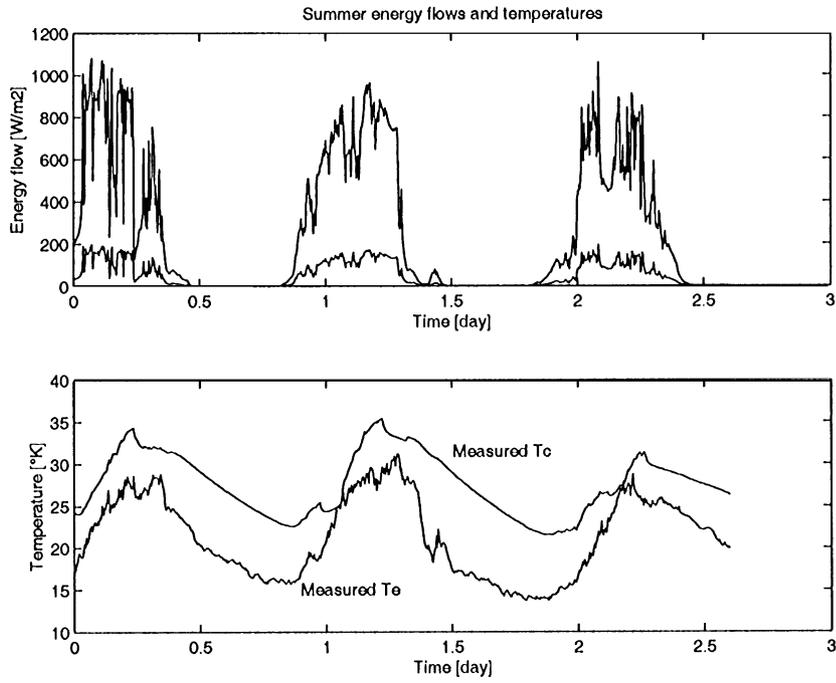


Fig. 15. The measured data in summer.

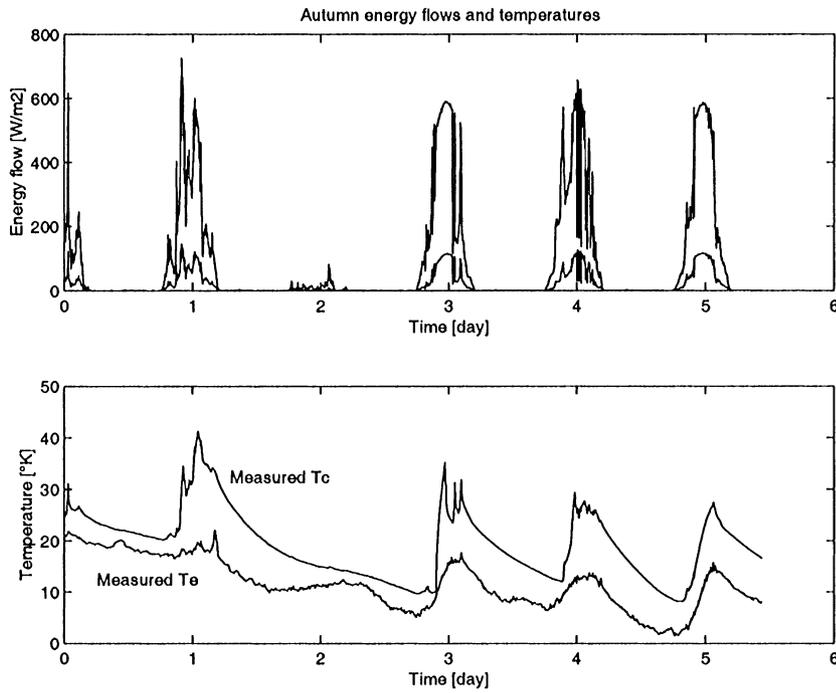


Fig. 16. The measured data in autumn.

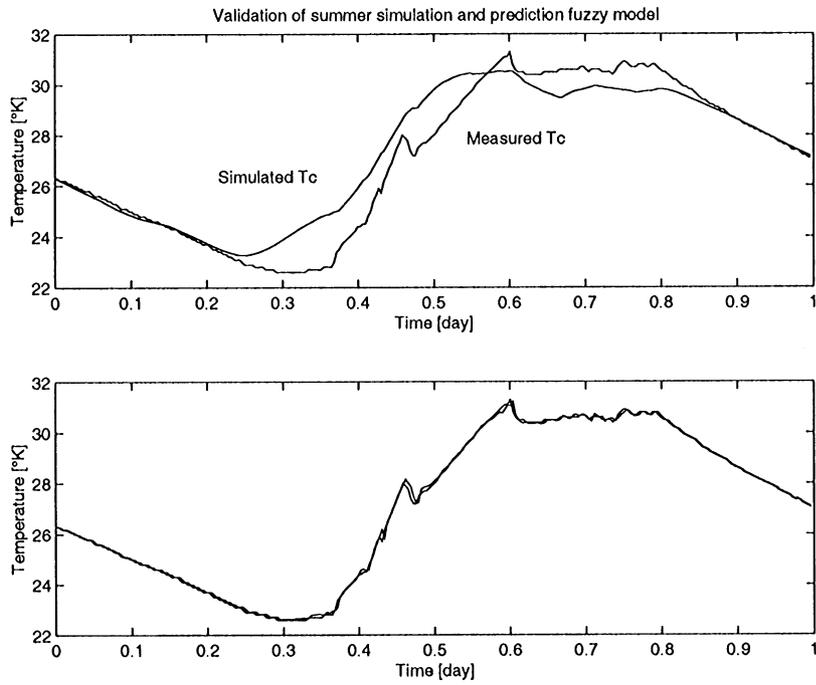


Fig. 17. Validation of fuzzy models for the summer.

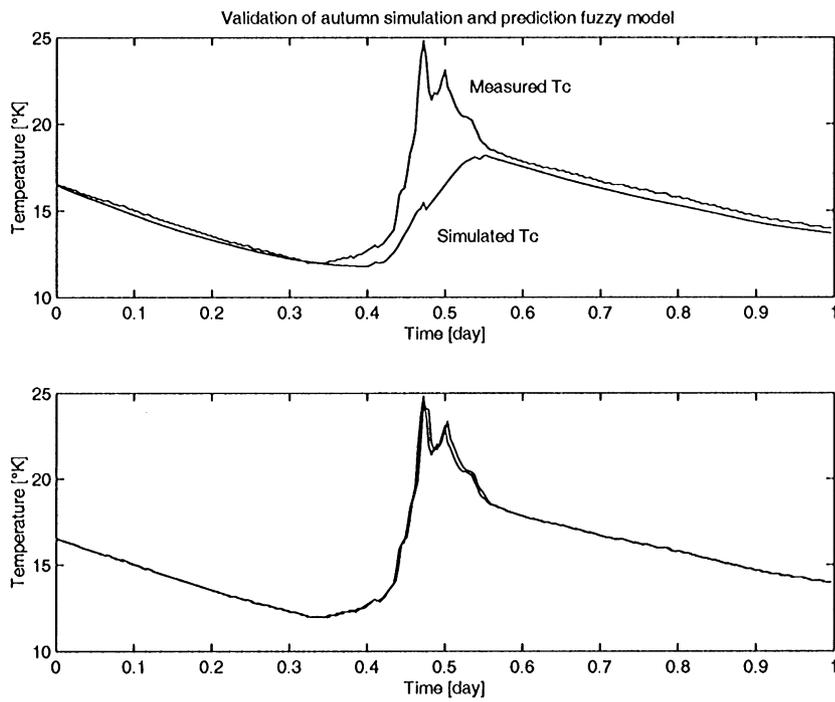


Fig. 18. Validation of fuzzy models for the autumn.

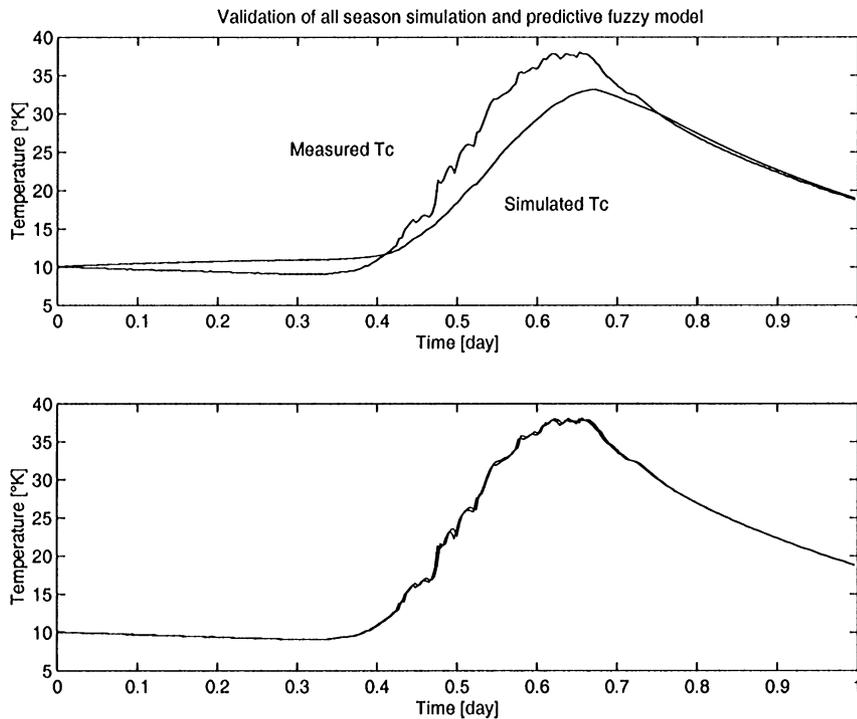


Fig. 19. Validation of all season fuzzy models.

the data collections, which were available only for two periods in the year, for summer and for autumn. These data sets were too small to develop a good whole year fuzzy model, but reasonable fuzzy models can be developed for each period. In spite of that, the results of fuzzy modelling are encouraging. The measured quantities of the thermal process are global solar radiation flow (q_s), outdoor temperature (T_e) and indoor temperature (T_c). Parts of the measured data are presented in Figs. 15 and 16.

In Fig. 17 the validation of the fuzzy model for summer period is given. The validation has been made on different data set. The upper part of the figure gives validation results of simulation model and the lower part the corresponding results of prediction model. In both cases the best results were obtained in case when both variables q_s and T_e have been divided into 3 membership functions.

In Fig. 18 the validation of the fuzzy model for autumn period is given. According to the autumn dynamics of the weather it is reasonable that the data demand would be greater as in the summer season. Due to the limited data set the results of the fuzzy modelling are reasonable. The upper part of the figure gives the validation results of the simulation model and the lower part the results of the prediction model. In both cases the best results were again obtained when both variables q_e and T_e have been divided into three membership functions.

Also the all season fuzzy model has been developed. It shows a good performance in summer season and a little bit worse in the autumn period. In Fig. 19 the valida-

tion of this model is given for the worst situation in autumn period. All comments about the quality of the fuzzy models are given for simulation type model. One-step ahead predictive-type fuzzy model gives much better performance for all periods of the year, and can be properly used for control purposes.

In further investigation process dynamics was modelled by the second-order fuzzy model structure. The results are in spite of more complicated structuring, almost the same as in the case of first-order fuzzy model.

4. Conclusions

The main purpose of the paper is to compare and discuss the main advantages and disadvantages of two different types of modelling, theoretical and experimental modelling. The theoretical modelling is based on energy balances, which gives the overall model expressed by differential equations. Those differential equations are then used in simulator called KAMRA. The main advantage of theoretical modelling is in parameter independent model, which can be used to simulate different plant with different parameters: so it is possible to simulate the behaviour in the case of different materials, orientations, geographic locations, positions and numbers of windows and periods of the year. The disadvantages of theoretical modelling are in complicated and time consuming procedures and in many simplifications, which are needed due to the

unknown behaviour of certain subprocesses in the thermal dynamics. The second type of the model is the experimental model, which is obtained on the basis of plant measurements. In this paper a fuzzy approach is investigated. Non-linear relations between input and output variables, which are obtained by least-squares optimisation method, represent the model. The non-linear fuzzy model, which is based on modified Sugeno structure, has two inputs (global solar radiation flow, outdoor temperature) and one output (indoor temperature). The main modification of Sugeno fuzzy model form is made in a sense that variables in antecedent part are not directly used in linear combination in consequent part of the rule but they influence the parameters of the consequent part. This gives us one-step ahead fuzzy predictive model. This type of model could be seen as a model, which has been learned on the basis of measurement, made on the plant. So it cannot give results as relevant as in the case when the inputs to the model are very different from those which are in the learning set. To obtain a good model for the whole working area, the additional measurements should be provided. This is the main disadvantage of this type of modelling. But it is not a real disadvantage in comparison to the theoretical model, which has to be validated in all regions in which it is used for simulation. So the appropriate measurements are needed also for theoretical modelling.

The different modelling approaches were investigated with different final model goals so the comparison was described in the sense of general advantages and drawbacks obtained from many different situations, measurements, simulation studies and experiences as well.

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